ACCURACY OF PREDICTION METHODS FOR SOUND REDUCTION OF CIRCULAR AND SLIT-SHAPED APERTURES

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Sound leakage caused by circular and slit-shaped apertures is well known to cause significant loss of sound reduction performance for doors, partitions, floor/ceilings and acoustic enclosures. A number of theories are available for predicting the diminished sound reduction of such apertures. In general, validation of these theories has been limited; the result of lacking a sufficiently extensive set of measurement data for comparison. This paper collates available, published sound reduction measurement data for circular and slit-shaped apertures. Comparisons are made with predicted sound reduction performance, determined using several of the available theories, to evaluate their accuracy.

Keywords: aperture, slit, leak, sound reduction

1. Introduction

Sound leakage caused by circular and slit-shaped apertures is well known to cause significant loss of sound reduction performance for doors, partitions, floor/ceilings and acoustic enclosures. This study collates 49 test measurements of aperture sound reduction from five relevant publications. The measurement results are compared to estimated sound reduction levels determined using three different prediction models. Comparison results are consolidated and reviewed qualitatively to understand the relative accuracy of the different prediction models used.

2. Referenced literature

Published literature concerning the sound reduction of circular and slit shaped apertures is relevant to the present study in two ways: informing prediction models, and; providing measurement data for comparison with predictions. A full list of all referenced literature is provided at the end of this paper. Key publications are noted in Table 1 below.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Author</th>
<th>Date</th>
<th>Prediction model</th>
<th>Measurement data</th>
</tr>
</thead>
</table>
Models for the prediction of aperture sound reduction are provided in a number of the referenced publications. As shown in the table, two models have been chosen in this study for comparison with measurement data: Gomperts (1964) [1] and Mechel (1986) [2]. Each of these models is briefly described in Section 3 below. A third, simpler model is also considered where aperture sound reduction is assumed to be zero across the assessed frequency range. This will be referred to as the Jones (1976) model.

Measurement data from five publications has been identified for comparison purposes. These publications are also identified in the table. From all five of these papers, measurement data for rectangular slits has been gathered. While some of the referenced publications provide measurement data for circular and other-shaped apertures, such results are limited in number. They are therefore excluded from the present study, which consequently only considers slit shaped apertures.

3. Models

As detailed in ISO 10140-2:2010 [8], sound reduction R is defined as:

\[ R = 10 \log_{10} \left( \frac{W_1}{W_2} \right) = -10 \log_{10} \left( \frac{W_2}{W_1} \right) \]  

Here \( W_1 \) and \( W_2 \) are the sound power incident on and radiated by the test element, respectively. In the following subsections, formulae are presented for the ratio of sound power levels in the form \( \tau = \left( \frac{W_2}{W_1} \right) \).

3.1 Gomperts (1964)

Gomperts [1] provides the following equation for the sound reduction performance of an infinitely long, empty rectangular slit exposed to a diffuse sound field:

\[ \tau = \frac{8k \cos^2(khe)}{2n^2 \left\{ \sin^2 \left( \frac{k}{h} \left( \frac{d}{h} + 2e \right) \right) + \frac{(kh)^2}{2n^2} \left( 1 + \cos \left( \frac{k}{h} \left( \frac{d}{h} + 2e \right) \right) \cos(kd) \right) \} } \]

Here \( k = \frac{2\pi f}{c_0} \) is the wavenumber in air, \( h \) (m) is the height of the slit, \( d \) (m) is the depth of the slit (which is equal with the thickness of the partition or element containing the slit) and \( n = 1 \) for a slit in the middle of a partition or \( n = 1/2 \) for a slit adjacent to an edge. Lastly, \( e \) is the end reflection given by the following equation [9] where \( \gamma' \) is Euler’s constant = 0.57722...:

\[ e = \frac{1}{\pi} \left( \ln \left( \frac{8}{kh} \right) - \gamma' \right) \]

3.2 Mechel (1986)

Mechel [2] provides an equation for the sound reduction performance of an empty, unsealed rectangular slit exposed to free field sound at a specific angle of incidence, expressed using the polar angle \( \theta_i \) and the azimuthal angle \( \phi_i \):

\[ \tau(\theta_i, \phi_i) = \frac{Z_0 \Re[Z_2]}{\cos \theta_i} \left| \frac{2A \sin c(k \frac{h}{2} \sin \theta_i \sin \phi_i)}{A(Z_1 + Z_2) \cos(k \cos \theta_2) + j[A^2 + Z_1Z_2 \sin(k \cos \theta_2)]} \right| \]
The slit is assumed to be infinitely long. Variables \( k, d \) and \( h \) are as noted above. \( A = Z_0 / \cos \theta_2 \) and \( \cos \theta_2 \) is the complex angle of refracted sound within the slit which can be determined from the relationship \( \cos^2 \theta_2 = \sin^2 \varphi_i + \cos^2 \theta_1 \cos^2 \varphi_i \). \( Z_0 \) is the characteristic impedance of air and \( Z_1 \) and \( Z_2 \) are impedances at the terminations of the slit which can be expressed as a weighted sum of Hankel functions. Mechel’s paper includes estimates for the Hankel functions expressed as polynomials. An estimate of the diffuse field aperture sound reduction can be calculated by numerically integrating the related double integral [4]:

\[
\tau_{\text{diffuse}} = \int_0^{2\pi} \int_0^{\pi/2} \tau(\theta_i, \varphi_i) \sin \theta_i \cos \theta_i d\theta_i d\varphi_i \\
\int_0^{\pi/2} \sin \theta_i \cos \theta_i d\theta_i
\]

\( (5) \)

### 3.3 Jones (1976)

Whereas the Gomperts (1964) and Mechel (1986) models estimate sound reduction values which are dependent on slit dimensions and frequency, the Jones (1976) model assumes that \( \tau = 1 \) at all frequencies such that the resulting sound reduction value is 0. While the Jones (1976) model may appear overly simplified in the context of aperture sound reduction, estimates with this model can be reasonable in the context of total sound reduction (being the combination of sound reductions through an aperture and its surrounding partition).

### 4. Prediction tolerances

A reasonable objective for aperture sound reduction predictions is to replicate laboratory measurements. This provides a bound on prediction accuracy, in the best case, of being equivalent to the accuracy of laboratory measurements. In this context, indicators of laboratory measurement accuracy become targets for the accuracy of predictions. Explicit guidance on laboratory measurements of aperture sound reduction, including accuracy requirements, is limited. Pragmatically, it is perhaps not surprising that there is no well established test standard or method as apertures in building elements are often unwanted leaks and gaps that occur inadvertently and under a broad range of circumstances that would seem not to be well suited to standardised laboratory conditions. Conversely, a robust, standardised measurement methodology could prove helpful analytically. For example, measurement data reviewed for the present study demonstrates a significant variation in aperture sound reduction results for a rectangular slit of common dimensions that was measured independently across three different studies (refer to Section 5 for further details).

In the absence of explicit guidance, the ISO 10140 series of standards provides detailed guidance on the laboratory measurement of sound insulation, with a general focus on physical building elements such as partitions and floor/ceilings. Some measurement advice concerning apertures is provided in Annex J (Normative) Joints filled with fillers or seals – Sound reduction index of ISO 10140-1:2016 [10], primarily addressing methods of installation and otherwise relying on reverberant-field based sound insulation measurement methods as detailed in ISO 10140-2:2010 [8]. Advice in ISO 15186-1:2000 [11] may also be relevant to aspects of aperture sound reduction measurements, but this standard has not been directly considered as part of the present study.

Guidance on assessing the uncertainty of ISO 10140 based laboratory measurements is provided in ISO 12999-1:2014 [12], including indicative one-third octave band uncertainty data for measurements as either: reproducibility; in-situ deviation, or; repeatability. These different conditions are described in Table 2. Standard uncertainty data from Table 2 of ISO 12999-1:2014 is reproduced graphically in Figure 1 below.

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[1] [4] states that a diffuse sound field can be represented by integrating for the polar angle over the range from 0° to 78° rather than the 0° to 90° range noted in Equation (5).
Table 2: – Uncertainty conditions

<table>
<thead>
<tr>
<th>Condition</th>
<th>ISO 12999-1:2014 definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repeatability</td>
<td>Condition of measurement that includes the same measurement procedure, same operators, same measuring system, same location…</td>
</tr>
<tr>
<td>In-situ</td>
<td>Condition of measurement that includes the same location and replicate measurements on the same object by different operators using different measuring systems</td>
</tr>
<tr>
<td>Reproducibility</td>
<td>Condition of measurement that includes different locations, operators, measuring systems…</td>
</tr>
</tbody>
</table>

Figure 1: ISO 12999-1:2014 Standard uncertainty data for airborne sound insulation

In the absence of specific standard uncertainty data for aperture sound reduction data, from the curves shown in the figure the ‘Reproducibility standard deviation with a coverage probability of 95%’ data are used through the remainder of this study as a ‘general indicator’ of the accuracy of prediction methods. In using this general indicator, the following points are noted:

- Aperture sound reduction measurements can involve elements with very small areas, such that seemingly small differences between actual and assessed element area could have a notable affect on measurement results. This may mean that the ISO 12999-1:2014 standard uncertainty data is relatively conservative in the context of aperture sound reduction.
- Conversely, the minimum standard uncertainty value for the chosen general indicator is 3.3 dB. In the context of aperture sound reduction, where values tend to lie in the range ±10 dB, a 3 dB variation could represent an arithmetic halving or doubling of the sound reduction value and could therefore be viewed as a less than conservative appraisal of variability.
5. **Comparison of predictions and measurements**

5.1 **Measurement data**

Key aspects of the measurement methodology used in each of the five reference publications are summarised in Table 3. In all cases, the rectangular slits were open, without absorptive infill and without sealant or filler of any kind. In broad terms, each set of data has been determined using one of two different measurement methods, as noted in the table. The sound reduction difference (‘S.R. difference’) method involved measuring the sound reduction of a partition with and without the slit exposed, relying on reverberant-field based sound insulation measurement methods such as those of the ISO 10140 series. The sound reduction values with and without the slit can be compared to estimate the sound reduction of the aperture element on its own [9]. This type of measurement method can be prone to influence from flanking noise, particularly when evaluating small apertures. The ‘Sound intensity’ method involved generating a reverberant sound field on the source side of the aperture and measuring the sound intensity levels on the receiving side of the aperture. This method could be expected to be less influenced by flanking noise effects.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Measurement method</th>
<th>No. of tests</th>
<th>Reported frequency range (Hz)</th>
<th>Range of dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>[2]</td>
<td>S.R. difference</td>
<td>24</td>
<td>100-6300</td>
<td>Height (mm)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Depth (mm)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Length (m)</td>
</tr>
<tr>
<td>[4]</td>
<td>Sound intensity</td>
<td>9</td>
<td>250-6300</td>
<td>0.5-8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.5-100</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.485-1.97</td>
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<td></td>
<td></td>
<td>50-70</td>
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<td></td>
<td>1.2</td>
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<tr>
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<td></td>
<td>75-100</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.8-9.8</td>
</tr>
<tr>
<td>[7]</td>
<td>Sound intensity</td>
<td>1**</td>
<td>100-5000</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>45</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8**</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>49</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

** [6] reports six measurement results for rectangular slits with a fixed height (1.5 mm), two different depths (75 mm and 100 mm) and three different lengths (3.8 m, 6.8 m and 9.8 m). It is understood that the measurements involved slits positioned in corners, in contrast to the other data sets considered in this study where the slits were positioned away from corners. The corner positioning of slits has been accounted for in the predicted aperture sound reduction values.

*** [7] reports two measurements results: for a rectangular slit with and without seals. Only data for the unsealed case is considered in the present study.

5.2 **Repeatability example**

Measurements by Uris et al [6] include tests of a slit with constant height and depth of 1.5 mm and 75 mm respectively and with three different lengths: 3.8 m, 6.8 m and 9.8 m. As the aperture sound reduction values considered in this study are independent of area, and the fundamental dimensions of the slit, the height and depth, are constant, the measured sound reduction values across the three lengths of slit could be expected to be equal. Variation in the results could therefore be taken as an indicator of the measurement variability under the repeatability condition. Relevant results are presented in Figure 2 below for reference.

5.3 **Reproducibility example**

Figure 3 below presents aperture sound reduction values from three different measurement sets [2, 4, 5]. In each case, the height and depth of the measured slit are common, being 2 mm and 50 mm respectively. As shown in the figure, there is notable variation in the measured mid and high frequency sound reduction values. Indeed, from the available data it is difficult to observe any consistency in the resonant frequency, which is theoretically a function of the common slit depth. Some of the variation in low frequency sound reduction may be due to differences in slit length, ranging from 500 mm to 1940 mm.
Figure 2: Aperture sound reduction measurements from [6], height 1.5 mm, depth 75 mm

Figure 3: Aperture sound reduction measurements from [2, 4, 5], height 2 mm, depth 50 mm

Figure 3 also presents predicted aperture sound reduction values using the Gomperts (1964) and Mechel (1986) models. It can be seen that the Mechel (1986) model is in good agreement with measured data from [4] whereas the Gomperts (1964) model provides better agreement with data from [2] and [5].
5.4 Overall accuracy

Comparisons have been made using all 49 sets of measured aperture sound reduction values and each of the three prediction models detailed in Section 3.0 above. These comparisons have been consolidated to provide a summary of general prediction accuracy and variance, with results presented in Figure 4 below. The range of assessed frequencies varies across the publications referenced for measurement data, as detailed in Table 3 above. Consolidated results have been analysed for the common set of assessed frequencies, spanning 250 Hz to 3150 Hz.

As shown in the figure, based on the available measurement data, the Gomperts (1964) and Jones (1976) prediction methods have mean sound reduction differences (predicted – measured) that lie within the nominated ‘general indicator’ tolerances. The Mechel (1986) model has mean sound reduction differences that are comparable to the ‘general indicator’ tolerances. Perhaps more importantly, however, the standard deviation for the mean sound reduction differences of the Jones (1976) model, as shown by the bar series plotted against the right hand side vertical axis, are notably larger than the equivalent standard deviations of the Gomperts (1964) and Mechel (1986) models. This indicates not only that the Gomperts (1964) and Mechel (1986) model are generally more reliable than the Jones (1976) model but also, given that the Jones (1976) model assumes a sound reduction of zero across the aperture, that there is significant variation in measured aperture sound reduction across the 49 test results. This variation has been noted as being particularly apparent across different measurement sets, as demonstrated in Section 5.3 above. In this context, the results of Trompette et al [4] warrant particular mention as they are generally in very good agreement with results of the Mechel (1986) model.

Figure 4: Consolidated results of comparisons of predictions and measurements

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2 The black dashed lines in Figure 4 show the Reproducibility standard deviation with a coverage probability of 95%
ACKNOWLEDGEMENTS

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REFERENCES